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Walrasian or Keynesian Equilibrium:
A Game Theoretic Approach

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Abstract

This paper presents a model of endogenous price formation where disequilibrium trading is possible. The purpose of the model is to examine whether an economy can get stuck away from Walrasian equilibrium. This is done by examining the class of Nash equilibria of the model. This class is very large and includes many inefficient, i.e. Keynesian, equilibria. When attention is restricted to a subclass of these equilibria, where price setters know the true consequences of altering their price signals, it is shown that they correspond to Walrasian equilibria and must exist in large economies.

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I. Introduction

The intent of this paper is twofold. First, we would like to construct a formal model in which the forces pushing an economy in disequilibrium are explained directly from individual optimizing behavior. Second, using this formal model, we would like to determine whether an economy can get stuck away from Walrasian equilibrium.

In order to accomplish our first purpose, our model must allow for actual trading in disequilibrium. The standard general equilibrium model, as characterized by Debreu's <u>Theory of Value</u> [ ], does not allow for disequilibrium trading. Neither do analagous Keynesian models such as Benassy [ ] and Dreze [ ]. In these Keynesian models, trades can only occur if the relevant quantity constraints are set at their equilibrium levels. Our model is a market game. Trading can occur without concern for any equilibrium consistency requirement. We restrict our attention to noncooperative play of the game in order to focus on competitive behavior. To achieve our second goal we will examine the Nash equilibria of our game and see if they coincide with Walrasian equilibria.

The model provides a rudimentary theory of price formation. The terms of trade are set by the agents themselves. In some sense our model generalizes the monopolistic competition model of Negishi [ ] and the conjectural equilibria models of Hahn [ ]. The key difference is that in our model all agents can set price. In these other models this capability is restricted to agents on one side of the market.

In Hahn's model this capability is further restricted so that only agents rationed at the prevailing price can signal their own price. Our view is that the ability to signal price is a technical rather than strategic capability. All agents should possess such a capability though signaling costs may vary across agents. On the other hand, the ability to command a certain volume of trade at a given price is a strategic capability which depends on the behavior of others.

When every agent can set a price there is no reason why a single price must prevail in any market. Our model has to describe how buyers and sellers are paired when there is dispersion of price signals. Two possible ways of doing this are as follows. The market may queue buyers and sellers on the basis of their price offers. Some examples of models of this sort are given by Dubey [ ] and Dubey and Shubik [ ]. Another possibility is that agents recognize each other via the price signal. This may entail some cost by either the price setter or the price taker. In this case, when a seller sets a price and a buyer wants to buy at that price, the buyer sends a buy signal directly to that seller. This second method is somewhat at odds with the general equilibrium model. There, agents are virtually anonymous. Each agent trades with an amorphous market. However, this second method is compatible with real world phenomena such as advertising and search. In addition, the first method rules out the possibility that infra-marginal traders on one side of the market are paired with extra-marginal traders on the other side of the market. This possibility should not be ruled out a priori when considering why economies may get stuck away from Walrasian equilibrium.

A major accomplishment of our model is that it incorporates endogenous price setting when agents are symmetric with regard to their signaling capabilities. The model is a two stage model. In all equilibrium models where agents are not distinguished by their signaling capabilities and in all the pricing games which we are aware of there is simultaneous signaling. In equilibrium models, agents choose their signals on the basis of observed market signals, e.g., prices and/or quantity constraits. These market signals are, in turn, generated by the joint signals of the agents. Equilibrium models do not come to grips with this apparent chicken-and-egg problem. In our model, we want to capture the intuition that, by altering his price signal, an agent can affect the volume of his trade. This agent must regard his price signal as informational input to others who contemplate trading with him. The agent must consider the relative merits of setting more favorable terms of trade versus inducing a greater volume of trade. The price setter acts as a monopolist (monopsonist). In the first stage of our model each agent sets prices. In the second stage agents signal trades at the prices already set.

Our model involves more elaborate behavior than is permitted in equilibrium models. Agents in the second stage signal trades to particular agents. The size of an agent's second stage signal increases with the number of agents in the economy. Note that we do not account for transaction costs in our model. When these costs are present it is likely that the optimal strategy for agents requires restricting the size of the second period signal. A more important point is that, when viewing our game in normal form, elements of an agent's strategy space include entire reaction functions. These functions determine the volume of trade signaled in stage two as the price signals in stage one vary. Since our strategy spaces are function spaces, analysis of the game is quite involved. There are technical problems which force us to illustrate existence of Nash equilibrium via construction rather than via the standard fixed point approach.

We have chosen to restrict our attention to a monetized economy. The primary purpose of money in the model is as a medium of exchange. All price signals are exchange ratios of goods in terms of money. This restricts the number of active markets.[1] An agent has to monetize his entire demand signal but the agent does not have to be concerned whether his cash will be accepted as a means of payment. This leads to an important question that must be resolved in disequilibrium models. What are the appropriate constraints on signaled trades? In disequilibrium, an agent may anticipate receipts which are, in fact, not forthcoming. If the agent does not have enough cash reserves to finance his already expressed demand, then the agent is bankrupt. Something must be done to resolve this difficulty.

An extreme Keynesian solution is to restrict signals so that the resulting trades yield feasible allocations regardless of which signaled trades actually occur. Under this type of constraint the value of an agent's demand signal is bounded by cash on hand. This is severe. In fairness however, the constraint is so severe only because it is cast in a static model. In a dynamic model where an agent has some choice over the timing of receipts and expenditures, such a constraint on instantaneous expenditures may not be very harsh.

In our model this type of contraint produces an additional difficulty. In any market both buyers and sellers will be setting a price. Price setting entails a commitment to a certain volume of trade. Suppose all agents set the Walrasian price in some market. What is to prevent any single agent the perception of market power? When a seller holds up on his sales at the Walrasian price, some unsatisfied demand should be created. This seller should perceive that there will be some demand at prices above the Walrasian price. In spite of sales at above the Walrasian price, the seller may desire to sell additional amounts at the Walrasian price. The more that is sold at the Walrasian price, the less will be demanded at the price the seller has set. Competitive equilibrium survives as an equilibrium of our model if, whenever the seller sells any amount at any price above the Walrasian price, he wishes to sell more than he actually is selling at the Walrasian price. Because the seller's sell signal at the Walrasian price occurs in stage two, he does not perceive an effect that this signal has on the demand at his price. However, because we are insisting on a Nash equilibrium solution, the buyers' demand at the seller's price is a function of how much the seller sells at the Walrasian price. In equilibrium there is no demand at the sellers price so the seller is, in effect, a price taker at the Walrasian price. Crucial to the existence of equilibrium is a residual volume of demand signal at the Walrasian price. The seller can fulfill his desired sales at the Walrasian price when he does not sell as much as he wants at the price he has set. When agents are restricted so that the total value of demand they have signaled, at both the prices they and others have set is bounded by cash on hand, this residual volume of demand signal may not be present. For an elaboration of this point see Arvan [ ].

We adopt another approach. We assume there is an institutional rule which alters actual trades when they would yield individually infeasible allocations. The rule does not affect trades when individual feasibility is already guaranteed. Note that such a rule alters trades of creditors of the bankrupt agent as well as the trades of the bankrupt agent himself. Though we do not concern ourselves with this issue here, the model seems to be appropriate for an investigation how, and under what circumstances, demand signals will be effective when they are not backed by ready cash. Such considerations may also provide a motive for financial intermediation. There are technical problems with showing existence of Nash equilibrium in the stage two subgame under such a rule. These problems arise because an agent's signaled trades at other agents' prices may affect the feasibility of someone else's allocation which, under this rule, may alter the trades that the original agent partakes in at the prices he has set. This indirect effect may be nonlinear. An agent may rationally perceive that his set of potential trades is not convex. This problem is not insurmountable. For a solution see Arvan [ ].

Because disequilibrium trading is permitted, signaled trades will not, in general, coincide with actual trades. We would like to view the rationing process itself as a competition. Agents spend scarce resources to promote their trading interests. In Walrasian equilibrium there is no reason to utilize resources for this purpose. When comparing allocations obtained under perfectly competitive versus disequilibrium models, we must make allowance for the fact that the spending of resources to promote trading is likely to have a nonneutral effect on the equilibrium of the economy. To avoid this problem in our model, we let actual trades be a function only of signaled trades. This lends an air of unreality to the model but it is consistent with the Keynesian, general equilibrium approach.[2] We do allow, however, for an agent to have some influence on his quantity constraint via his own quantity signal.[3]

II. The Formal Model

Consider an economy with K nonmonetary goods indexed by k. A bundle of goods is represented by x. There are N agents indexed by n. Each agent has utility $u^n(x,m), u^n\colon\ \mathrm{IR}_+^{K+1} \to \mathrm{IR}$, where m is an amount of the monetary commodity. Assume u^n is strictly quasiconcave and increasing for each n. Each n has an endowment of goods, $w^n = (w_1^n,\ldots,w_K^n)$, and an endowment of money, m^n. Assume $w^n \in \mathrm{IR}_+^K$ and $m^n \in \mathrm{IR}_+$. Trade is not necessary for survival. Call all such economies classical exchange economies.

Now consider the following price game. There are two stages in this game. Stages are indexed with either a one or two. The idea is that stage two is "later" than stage one. During stage one, agents set prices for the nonmonetary goods in terms of the money commodity. Agent n's action during stage one is $\alpha^n = (p^n,q^n)$ where $p^n \in \mathrm{IR}_{++}^K$ and $q^n \in \{1,-1\}^K$. These symbols have the following interpretation. p_k^n is the price n sets for good k. If $q_k^n = 1$ then n is setting a buy signal for good k. If $q_k^n = -1$ then n is setting a sell signal for good k. Let $\alpha = \underset{n}{X}\alpha^n$. In stage two, α is public knowledge.

In stage two agent n sends the price taking signal $\beta^n = (s^n, t_1^n, \ldots t_N^n) s^n \in \mathbb{R}^K$. $s_k^n > 0$ only if $q_k^n = 1$. $s_k^n < 0$ only if $q_k^n = -1$. $t_i^n \in \mathbb{R}^K$ for each i and $t_n^n \equiv 0$. t_i^n is the quantity signal n sends to agent i. $t_{ik}^n > 0$ only if $q_k^i = -1$. $t_{ik}^n < 0$ only if $q_k^i = 1$. In words, n signals to buy from i units of $k(t_{ik}^n > 0)$ only if i signals to sell units of $k(q_k^i = -1)$, and vice versa. Let $\beta = \underset{n}{X}\beta^n$, $\beta/n = \underset{i \neq n}{X}\beta^i$, $t_i = \sum_n t_i^n$, $t = \underset{i}{X} t_i$, $t_i/n = t - t_i^n$ and $t/n = \underset{i}{X} t_i/n$.

The next step is to describe transactions. Assume individual feasibility is not an issue. Let y_k^n be the actual transaction made by agent n as a price setter for good k. The rules governing y_k^n are:

$$
\begin{aligned}
y_k^n &= \min(s_k^n, -t_{nk}) \text{ if } s_k^n > 0. \\
&= \max(s_k^n, -t_{nk}) \text{ if } s_k^n < 0. \\
&= 0 \qquad\qquad\quad \text{ if } s_k^n = 0.
\end{aligned}
$$

This is the most obvious rationing rule. Each price setter is a monopolist (monopsonist) for the good that he has set the price. As such, the magnitude of his trade is the minimum of supply and demand.

For price takers, the actual transaction agent n makes with price setter i is z_i^n. The rules governing z_i^n are a little harder to formulate, because the trades must be split among all price takers. Consider the class, $\mathcal{T}$, of rationing rules specified by the following properties. $T \in \mathcal{T}$ and write $z_{ik}^n = T(s_k^i, t_{ik}/n, t_{ik}^n)$. Then:

(1) $z^n_{ik}\ t^n_{ik} \geq 0$

(2) $|z^n_{ik}| \leq |t^n_{ik}|$

(3) When $t^n_{ik} > 0$, i) T is nondecreasing in t^n_{ik} and ii) T is decreasing in s^i_k, t_{ik}/n.

(4) When $t^n_{ik} > 0$, i) T is concave in t^n_{ik}, and ii) T is convex in t_{ik}/n.

(5) T does not favor buyers or sellers, i.e.,
$T(s^i_k, t_{ik}/n, t^n_{ik}) = -T(-s^i_k, -t_{ik}/n, -t^n_{ik})$

(6) T is continuous

(7) Price taking transactions are "consistent" with price setting transactions, i.e.,
$y^i_k = \sum_n z^n_{ik}$. In particular
$z^n_{ik} = t^n_{ik}$ if $|y^i_k| < |s^i_k|$.

(8) T is nondiscriminatory. If $t_{ik}/n = t_{ik}/j$ and $t^n_{ik} = t^j_{ik}$, then $z^n_{ik} = z^j_{ik}$.

Note that the nondiscriminatory property is not assumed by Benassy or Dreze. The resulting commodity bundle for agent n after stages one and two is:

$$(w^n + y^n + \sum_i z^n_i,\ m^n - p^n y^n - \sum_i p^i z^n_i). \quad \text{Note that } z^n_n \equiv 0.$$

To complete the description of our game we must specify the strategy spaces. This amounts to specifying constraints on the stage two signals. The constraints on agent n in stage two are given by (9) - (14) below.

(9) $-s_k^n \leq w_k^n \ \forall k$

(10) $\sum_k p_k^n \max(0, s_k^n) \leq C_s^n(\alpha)$

(11) $\sum_{i \neq n} \sum_k - \min(0, t_{ik}^n) \leq w_k^n$

(12) $\sum_{i \neq n} \sum_k p_k^i \max(t_{ik}^n, 0) \leq C_t^n(\alpha)$

(13) $s_k^n q_k^n \geq 0 \ \forall k$

(14) $t_{ik}^n \, q_k^i \leq 0 \quad \forall i \neq n, k$

The set of all permissable stage two signals will be denoted as $B^n(\alpha)$.

<u>Discussion</u>: These constraints do the following. The set of signals allowed is compact. If the transaction rule is continuous this means that given β/n an optimal signal will exist for agent n. Furthermore, if trades are increasing in signals then bounding signals is necessary for there to be an optimal signal in equilibrium with rationing. An objection may be raised that when the transaction rule itself generates feasible allocations, agents should not be constrained at all in their signals. Two potential interpretations of these constraints mute this criticism. First, signaling itself may be costly. These costs, e.g., search and advertisement costs, do influence the extent of trade in actuality. The signal constraints can be taken as a proxy for these

costs. While it may be more realistic to model such costs explicitly, doing so, in effect, introduces new commodities and thereby alters equilibrium. The constraints as specified do not. This makes it easier to compare equilibria under various assumptions about information and strategy. The second interpretation has to do with penalties due to bankruptcy or short selling. Such penalties should deter overly optimistic signaling in a world of imperfect or incomplete information. The signaling constraints can be thought of as proxies for these penalties. In this case, however, since bankruptcy and equilibrium are not compatible, there is less justification for wanting a proxy rather than explicit penalties. However, in a world where individuals have positive subjective probabilities of defaulting on their commitments and where gains from trade are bounded for the individual, one would expect the magnitude of optimal signals to stay within certain limits.

As to the constraints themselves the following should be noted. Stage one signals do not impinge on stage two signals except through the consistency constraints, (13) and (14). In both price setting and price taking, supply signals have no influence on the demand signal constraint. The number $C_s^n(\alpha)$ is presumed to be large enough so that Walrasian trades can be signalled. A possible value of $C_s^n(\alpha)$ is the value of n's endowment capitalized at the prices he has set. A similar interpretation can be made for $C_t^n(\alpha)$. Supply signal constraints "force" the price taking seller, who is rationed in trading with agent i, to reduce his supply signal to agent j. Rationing forces agents to "spend" their signals. Hence, an optimal signal is determined by both the transaction rule and the signal constraints. This is both a departure and a generalization of the fixed quantity constraint models. The constraint on signals "causes" the monopolist's demand curve to be more elastic than it otherwise would be at the competitive equilibrium price.

Now we must account for individual feasibility. Below, z^n_{ik} is the actual trade after the feasibility rule is applied. $y^n_k = -\sum_{i\neq n} z^i_{nk}$. The feasibility rule is given by the following problem

$$\underset{\{z^n_{ik}:\ n=1,\ldots,N;\ i=1,\ldots N, i\neq n;\ k=1,\ldots,K\}}{\text{maximize}} \quad -\sum_{n,i\neq n,k} (z^n_{ik}-t^n_{ik})^2$$

subject to:

(15) $z^n_{ik}\, t^n_{ik} \geq 0\ \forall n, i\neq n, k$

(16) $|z^n_{ik}| \leq |T(s^i_k, t_{ik}/n, t^n_{ik})|\ \forall n, i\neq n, k$

(17) $-\min(q^n_k, 0)\ [\max(s^n_k, -t_{nk}) + \sum_{i\neq n} z^n_{ik} + w^n_k] \geq 0\ \ \forall n,k$

(18) $m^n - \sum_{i\neq n} (p^i \cdot z^n_i - p^n \cdot z^i_n) \geq 0\ \ \forall n.$

The constraint set is nonempty if the original endowment is feasible. Then the zero trade vector satisfies all the constraints. The constraint set is closed and convex since all constraints are weak linear constraints. The family of constraints denoted (16) guarantees boundedness of the constraint set. The objective function is strictly concave. Consequently, there is a unique maximum so the trade rule is well specified. For α fixed, the constraint set is continuous in β since T is a continuous function and since each constraint is linear in the vector z. Thus, trades are continuous in β by the Berge maximum principle.

If the family of constraints denoted by (17) and (18), the "feasibility constraints," are satisfied by $z^n_{ik} = T(s^i_k, t_{ik}/n, t^n_{ik}) \forall n, i \neq n, k$, then the trade rule coincides with the previous scheme.

This is a complete description of our model. We proceed to solutions. Let $\beta/n(\beta\ )$ be the aggregate price taking signal excluding agent n, when the joint price taking signal is β. Let $B^{n*}(\alpha,\beta/n)$ be the optimal stage two correspondence of agent n given α and β/n.

Definition 1: For fixed α, β^* is a Price Taking Nash Equilibrium, P.T.N.E., if $\beta^{n*} \in B^{n*}(\alpha,\beta/n(\beta^*))\ \forall n$. Let $B^*(\alpha)$ be the set of all P.T.N.E. given the stage one signal α.

Theorem 1: $B^*(\alpha) \neq \phi$.

Proof: See Arvan [ ].

Theorem 2: The correspondence $B^*(\quad)$ is u.s.c. in α.

Proof: See Arvan [ ].

The next step is to proceed to solutions of the overall game. A strategy in the overall game for agent n is a stage one signal, α^n, and a function $\beta^n(\quad)$, which chooses a stage two signal given the joint period one signal such that $\beta^n(\alpha) \in B^n(\alpha)$ for all possible α.

Definition 2: A Nash equilibrium of the overall game, N.E., is a vector α^*, and a vector of price taking functions,

$$\beta^*(\quad), \ni \forall n,\ v^n(\alpha^*, \beta/n(\beta^*(\alpha^*)), \beta^{n*}) =$$

$$\sup_{\alpha^n, \beta^n(\ )} v^n((\alpha^*/n, \alpha^n),\ \beta/n(\beta^*(\alpha^*/n, \alpha^n)),\ \beta^n(\alpha^*/n, \alpha^n)).$$

v^n is the indirect utility function of agent n.

Note: At a N.E., $\beta^{n*}(a^*) \in B^{n*}(\alpha^*, \beta/n(\beta^*(\alpha^*))$. In other words, at a Nash equilibrium of the overall game, the price taking functions applied to the stage one signals give rise to a P.T.N.E.

The class of Nash equilibria is so large that for any stage one signal, $\bar{\alpha}$, there is a corresponding N.E. This N.E. is constructed as follows. Let $\bar{\beta}(\bar{\alpha}) \in B^*(\bar{\alpha})$ and for all n let $\beta/n(\bar{\alpha}/n, \alpha^n) = 0$ if $\alpha^n \neq \bar{\alpha}^n$. In this N.E. when agent n alters his price signal, no trade occurs. Furthermore, in the P.T.N.E. at $\bar{\alpha}$ each agent does at least as well as at his endowment, since no trade is always a possible second stage signal.[4]

It is debatable whether the existence of these N.E. gives credence to the Keynesian view. When interpreting the function $\bar{\beta}/n$ as n's stage one expectation of everyone else's stage two signal, these equilibria exist only because agents have "unreasonable" expectations about trading opportunities at other than equilibrium prices. It is difficult to give meaning to the "reasonableness" of expectations. However, we can examine the extreme case of rational expectations. Within the context of this model, the rational expectations case is equivalent to subgame perfection of the equilibrium. A subgame perfect N.E. is a N.E.,

(α^*,β^*), such that $\beta^*(\alpha) \in B^*(\alpha)$ for all α. Note that there always exists a trivial P.T.N.E., $\beta = 0$. When price setters signal a zero volume of trade there is no reason for price takers to signal other than zero and hence no reason for price setters to signal other than zero. The rest of the paper is devoted to studying the nature and existence of nontrivial, subgame perfect N.E.

III. The Pricing Game for K = 1

In this special case it is a simple matter to show that a competitive equilibrium, C.E., can be supported by a subgame perfect N.E. Let p^* be a C.E. price. If each agent signals p^* in stage one then all nontrivial P.T.N.E. at this stage one signal yield the C.E. allocation. Suppose an agent sets a price that is more favorable to him than p^* while everyone else signals p^*. There is a P.T.N.E. where no agent trades with this original agent and the C.E. allocation is attained. In this P.T.N.E. the original agent acts as a price taker at the C.E. price. In this equilibrium no agent has monopoly power.

A more interesting question to ask is the following. Do there exist P.T.N.E. where a seller, j, signals $p^j > p^*$ and where j does better than by pricing at p^*? Sometimes such P.T.N.E. exist. This is illustrated in the case N = 2 via an Edgeworth box diagram. Suppose agent 1 is the buyer and agent 2 is the seller. We assume that each n signals $s^n = x_n(p^n)$, where x_n is the net, excess demand function of agent n. The price setting signals p^1 and p^2 determine line segments from the endowment point with negative slope. These segments are such that the projection onto the x axis has length $|s^n|$ n=1,2 while the projection onto the y axis has length $p^n|s^n|n=1,2$. Given (p^1,s^1) and (p^2,s^2) the set of all possible trades yields the allocations which lie in the parallelogram, including interior, determined by these segments.

Figure 1

A P.T.N.E. yields an allocation in the parallelogram where further movement by agent 1 in the direction of 2's price offer makes 1 worse off. If 1 is buying at p^2 it must also be that a movement in the opposite direction of 2's price offer also makes 1 worse off. 2 must be in a similar position with regard to 1's offer. A P.T.N.E. yielding allocations in the interior of the parallelogram must be on 1's income expansion path at p^2 and on 2's income expansion path at p^1. It is possible that there are P.T.N.E. where 1 buys nothing at p^2 or where 2 sells nothing at p^1. These boundary P.T.N.E. do not necessarily correspond to intersections of the expansion paths. For instance, if 2 sells nothing at p^1 in a P.T.N.E., then it may be that 2 has positive net demand at p^1.

It is possible for there to be 1) only a single boundary P.T.N.E. allocation, 2) both interior and boundary P.T.N.E. allocations, and 3) only interior P.T.N.E. allocations. Figure 2 illustrates the case with both boundary and interior P.T.N.E. allocations. The figure demonstrates that the P.T.N.E. correspondence is not necessarily convex and need not be l.s.c. This last point is illustrated at the interior P.T.N.E. allocation which is a tangency point of the two income expansion paths, $I^1(p^2)$ and $I^2(p^1)$.

In the general case with $K \geq 1$, a proof of the existence of a subgame perfect N.E. using a standard fixed point argument is not appropriate. In fact no such equilibrium may exist. With more than one nonmonetary good, a seller setting a price higher than the equilibrium price may force buyers to buy from him at his price by "seeding" demand, i.e, by offering a slightly lower than equilibrium price in a complementary market.

Figure 2

A different approach is taken here. Seeding as a pricing strategy is rational only if, when looking at the market where j charges above the equilibrium price, there exists a P.T.N.E. where j does better than at the competitive equilibrium. It will be argued that as the market share of any agent falls, via replication of the economy, the power to act monopolistically in the above sense also falls. At some market share level greater than zero, no agent has any monopoly power. The market share level where this occurs depends on the utility functions and endowments of all agents in the original economy. In the case where all goods are gross substitutes, seeding is not a rational strategy. No agent can act monopolistically even if the agent is the sole seller of a particular good.

Assume agent 1 sets $p^1 = p*$ and $s^1 = x_1(p*)$ while agent 2 sets $p^2 > p*$ and $s^2 = x_2(p^2)$. In this case one P.T.N.E. is $t_2^1 = 0, t_1^2 = x_2(p*) = -s^1$. This is the P.T.N.E. which supports the C.E. If $I^2(p*)$ intersects $I^1(p^2)$ within the parallelogram, as in Figure 3, there is a P.T.N.E. where 2 does better than he does in the C.E. This intersection point is further out than the C.E. allocation on $I^2(p*)$. There may also be a boundary P.T.N.E. where agent 2 sells nothing at p*. Raising p^2 shifts the $I^1(p^2)$ curve up continuously. The $I^2(p*)$ curve cannot have slope less than or equal to -p*. It is evident that when there are interior P.T.N.E., agent 2 can raise p^2 further to yield a boundary P.T.N.E.

Figure 3

It is easy to see that at such a boundary P.T.N.E. allocation, 1's offer curve must lie on or below $I^2(p^*)$. In fact, this must also be true for an interior P.T.N.E. Figure 4 presents an inconsistent picture showing why this must be the case. Suppose, in Figure 4, that the segment from w to A represents the effect of 2 selling at p*

Figure 4

when the final allocation ends up at B. Let C be the intersection of segment AB and 1's offer curve. If 2 signals as indicated, then 1 can obtain point C. The segment wC is less steep than the segment AB. Hence at the price for which C is optimal for agent 1, B is affordable. Consequently C is preferred by 1 to B. Therefore B cannot lie on $I^1(p^2)$. The intersection of $I^1(p^2)$ and $I^2(p^*)$ must lie outside the parallelogram. It is evident that when 1's offer curve lies below $I^2(p^*)$ that a P.T.N.E. exists where 2 sells nothing at p^*.

Notation: Let E be an exchange economy and rE a replication economy of E. Denote by i_h the hth buyer of type i and by j_h the hth seller of type j, h = 1,2,...r.

Theorem 3: Let E be a classical exchange economy for K = 1. Let p^* be a C.E. price for E and let $\bar{p} > p^*$. Assume each agent's utility function is differentiable in a neighborhood of his C.E. allocation. Then there exists a natural number $\bar{r}$, such that for $r \geq \bar{r}$, in the economy rE if any seller j_h signals $p = \bar{p}$ while all other agents set price at p^*, then in no P.T.N.E. does any agent buy at $\bar{p}$.

The intuition behind the theorem is as follows. As a result of the curvature and nondiscriminatory properties of the rationing rule, all agents approximately attain their C.E. allocations as r gets large. Since utility functions are continuously differentiable near the C.E. allocations, individual demands at above the C.E. price beocme zero for r large enough.

This result does not generalize if some type of buyer i has "kinked" indifference curves. In this case it is possible to construct examples where buyers have nonzero demand at $\bar{p}$, even for large r, and where the aggregate demand at $\bar{p}$ is bounded away from zero for all r. These examples are pathological in the sense that the C.E. allocation for these buyers must be at a kink. These kinks occur on a closed set with empty interior in the consumption set. However, it should be noted that when such pathologies arise, schemes such as proportional rationing do not make the aggregate demand at $\bar{p}$ converge tc zero.

If a seller perceives gains by deviating from p* in replication economies, it is evident that these gains must be small and are decreasing in r. In fact, for sufficiently large economies, no gains exist. This is true because the slope of the "residual offer curve" approaches -p* as r gets large. In large economies, the "residual offer curve" lies above the deviating agent's income expansion curve. This follows because a large number of agents must be rationed at the C.E. price. Since these agents approximately attain their C.E. allocations, their demand at above the C.E. price is determined principally by the substitution effect. The following theorem results.[5]

Theorem 4: Let E be a classical exchange economy for K = 1. Let p* be a C.E. price for E. Assume the utility functions of all agents are twice continuously differentiable in a neighborhood of the C.E. allocation. Then there exists a natural number $\bar{r}$, such that for $r \geq \bar{r}$, in the economy rE, if a single agent, n, sets a price different from p* while all other agents signal p*, then in no P.T.N.E. does n do better than in the C.E. at p*.

IV. The Pricing Game for Arbitrary K

We would like to show that a result analagous to theorem 4 holds. We must show that seeding ceases to be profitable in large economies. The key to the result is that an agent cannot seed in a discriminatory manner. The effect of seeding on any particular agent gets small as the economy gets large. To see how the result is shown, imagine that agent n seeds in some market. Now look at any other market and view it as an economy with $K = 1$. Seeding can be thought of as creating a shift parameter in the utility functions. Since this shift effect gets small in large economies, the P.T.N.E. of these seeded economies converge uniformly to the P.T.N.E. of the unseeded economy. Theorem 4 holds for this unseeded, one market economy. Hence, seeding is less profitable than nonseeding. Our main theorem is:

Theorem 5: Let E be a classical exchange economy for arbitrary K. Let p^* be a C.E. price vector for E. Assume the utility functions of all agents are twice continuously differentiable in a neighborhood of the C.E. allocation. Then there exists a natural number $\bar{r}$, such that for $r \geq \bar{r}$, in the economy rE, if a single agent n sets a price different from p^* while all other agents signal p^*, then in no P.T.N.E. does n do better than in the C.E. at p^*.

Corollary: If all the conditions of theorem 5 are satisifed then, for the economy rE, a subgame perfect N.E. exists which supports the C.E. at p^*.

V. Extensions

The results we have obtained do not resolve the Keynesian-Walrasian debate. However, a framework was created which suggests several issues whose solution should shed light on the matter. Most of these solutions are likely to be extremely difficult to attain. We merely list the issues here.

First, we restricted our attention to P.T.N.E. Given the price signals, buyers and sellers matched themselves optimally. If we concerned ourselves with this matching problem, we would be forced to recognize that bottlenecks can occur even if prices are set at their Walrasian levels. The possibility of bottlenecks creates monopoly power. Is such power transient or is it destabilizing?

Second, our model is static. A dynamic version is needed. In a dynamic version we must address the possibility that current signals are sent not just to promote current trade but also to influence future behavior. We must come to grips with the question, how rational are our agents? We must also concern ourselves with issues such as stock valuation, borrowing and lending, and long term contracting.

The most promising unexplored area seems to be the incorporation of signaling costs into the model. A static framework can be maintained for this purpose. In particular, it would be of great interest to see if nonWalrasian allocations are supportable by nontrivial, subgame perfect equilibria, when signaling is costly.

Footnotes

[1]This point is discussed in Shapley-Shubik [ ]. In a barter economy out of equilibrium, an agent may be willing to offer apparently inconsistent exchange ratios. This inconsistency is accounted for by the perception that some goods will be more acceptable as a means of payment.

[2]Both Benassy [ ] and Dreze [ ] generate quantity constraints as a function of the joint excess demand signal.

[3]For example, a proportional rationing scheme is possible under our model.

[4]In an earlier version of this model, Arvan [ ], agents committed themselves to a volume of trade in stage one. In this formulation, the agent who alters his price could not be shut out by the other agents. This restricted the class of N.E. somewhat. However, this formulation had to be rejected due to difficulties with existence of P.T.N.E.

[5]Rigorous proofs of theorems 4 and 5 can be found in Arvan [ ].

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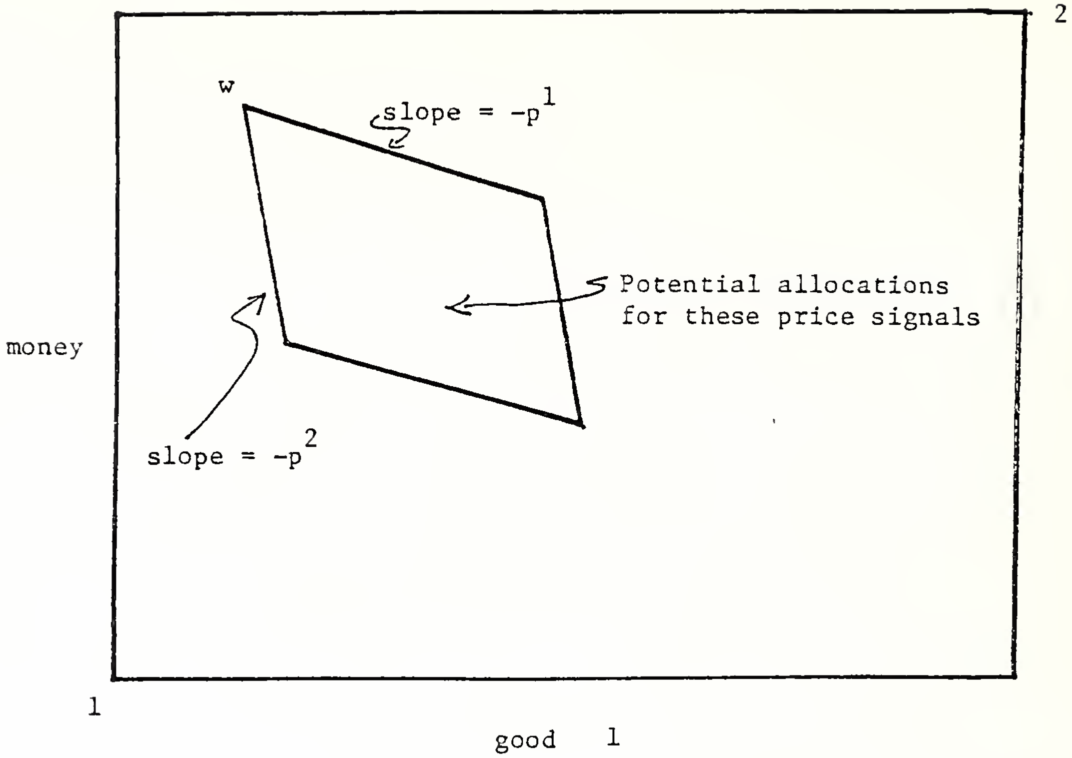

Figure 1

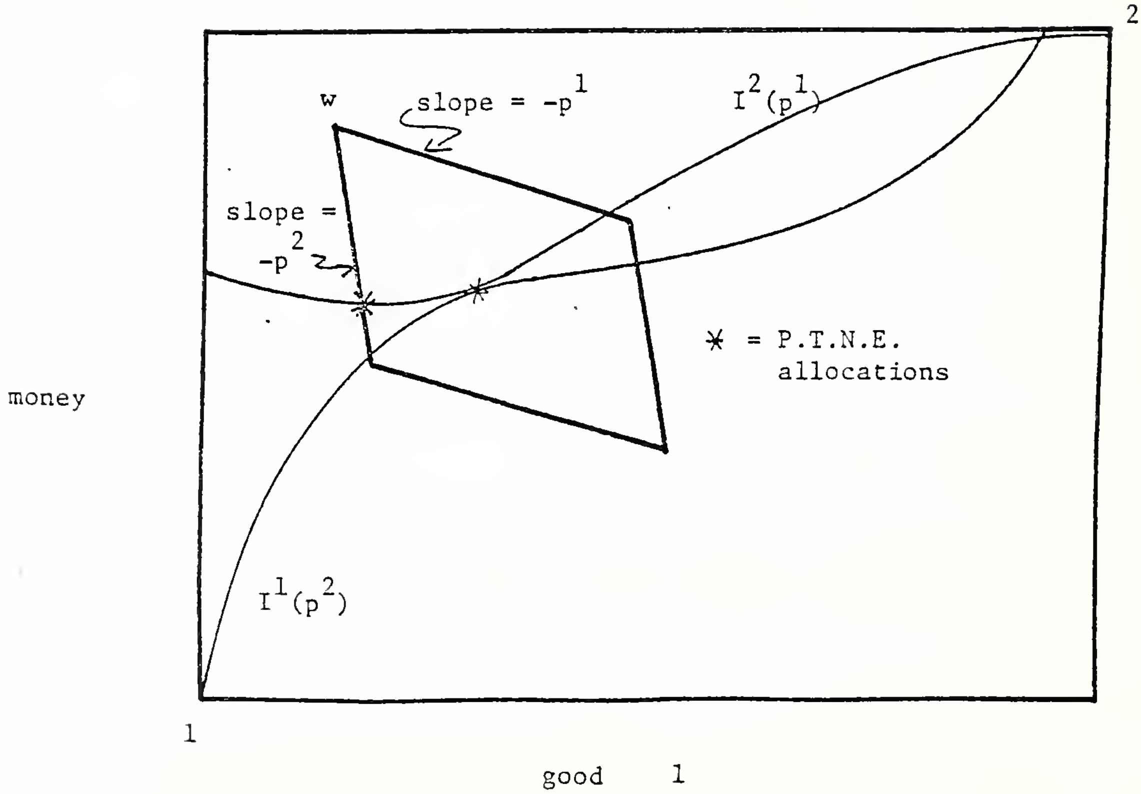

Figure 2

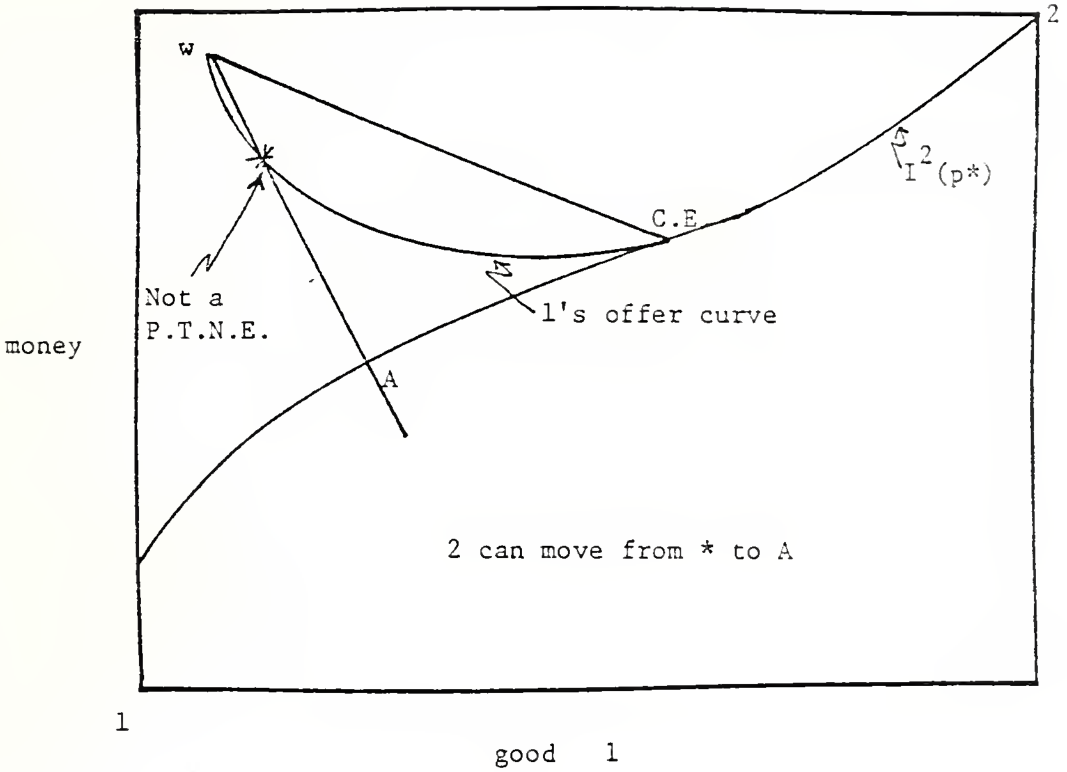

Figure 3

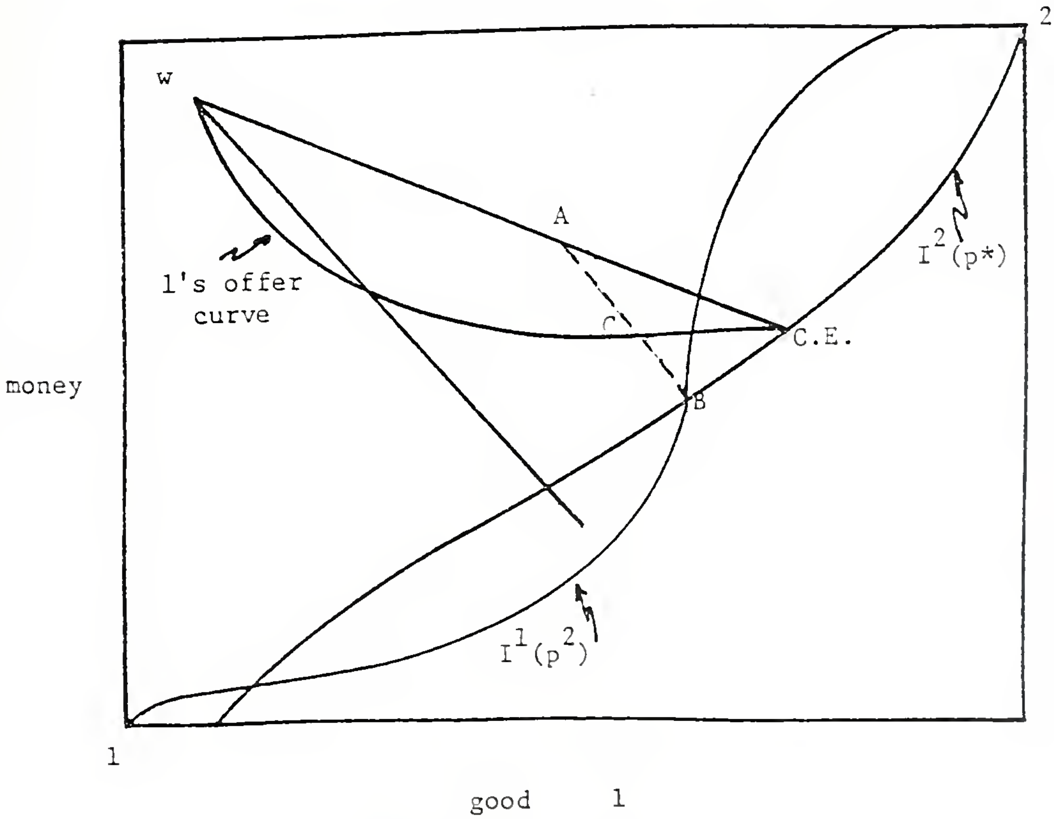

Figure 4